# The Relevance of Market Correlation for the Portfolio Selection – An Individual Investor Long Term Perspective

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ABSTRACT: This paper applies BEKK-type model to explain the interdependence between markets that might be relevant to a Polish individual saving for retirement. The investor is assumed to look for cross-country and cross-asset diversification of the long-term investment yielding the optimal portfolio, with performance assessed using local currency returns. Monte Carlo simulation methodology was used to better capture the market dynamics, especially with respect to market interdependence assumptions.

The high level findings of the optimal portfolio composition reconfirm that even a local currency investor, looking at local currency returns, can benefit from broadening of the investment spectrum, at the same time however providing supportive arguments for a strong bias towards local currency government bonds. The results question relatively high proportion of equity component offered as default by majority savings providers or mutual funds. This encourages reflection on the society-wide consequences of following predefined structures, frequently supported by policy reforms, which may lead to excessive shift of business risks onto the households. These results prove robust when using various measures of market interdependence as well as when capturing the recent COVID related market turmoil.

**JEL classification:** D53, G11, G17, G5, D14.

**Keywords:** Passive Investment Strategies, Pension Savings, Long-Term Investment, Home-Currency Bias, Market Interdependence.

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# 1 Introduction and background

Passive investment strategies have been growing in popularity over the last decade (Sushko and Turner, 2018). Increasing popularity of such investment strategies, first observed in developed markets (Gutiérrez and Philippon, 2016), has been gradually reaching converging economies, including Poland. At the same time, private savings of Polish households have more than doubled over the last decade (Analizy Online, 2020). Most traditional forms (bank deposits and cash) still dominate the individual portfolios, accounting for approx. 80% of the total savings. This form of investments has been gaining importance even during the low interest rate environment. This poses a question as to why alternative investment vehicles fail to offer an attractive alternative.

The increased volumes of individual savings may translate into capital critical for financing domestic investments. Therefore, understanding the determinants of optimal investment of private savings remains crucial not only from the point of view of individual savings but also from the perspective of economic development. While the first optic increases importance of optimising the investment decision (Jacobs et al., 2014), the second is a policy question, whether a market intervention is justified.

The question of an optimal investment portfolio is also important for decision makers (Walden, 2015). Institutional solutions always impose certain restrictions on investments. Introduction of investment vehicles, de facto formally endorses a particular investment strategy Thus one cannot ignore certain level of state responsibility for the outcome. The chosen investment strategy may both affect the society welfare (by influencing the value of pension savings and determine the risk transfer directions) as well as – to some extent – situation on the local capital markets. As argued in the literature (Beshears et al., 2009), even the theoretically voluntary character of such schemes does not provide an excuse from responsibility. When designing or amending regulatory solutions imposed on participants in the system, it is therefore critical to understand how close the portfolio, in fact, is to the model. In this context, the study attempts to answer the question of the optimal portfolio composition for an individual investor, who – believing in the theory of financial market efficiency, decides to maximize the Sharpe ratio (Sharpe, 1966).

This paper puts specific methodological focus on capturing the market interdependence when constructing the optimal portfolio. To this end, the study discusses the practical choices made when deciding on assumptions of the portfolio optimisation model. While the understanding of market dynamics and the tools for analysing it has developed massively over the last two decades (see, e.g., Tsiaras and Simos, 2020; Patra and Panda, 2021), they do not seem to be sufficiently reflected in the methods used for constructing optimal portfolios, especially for individual savings. The literature on the latter typically relies on very simplistic assumptions on the market volatility and correlation. This poses a question whether such methodological simplification remains justified and whether incorporating a more sophisticated approach to volatility and correlation could further optimise individual investment decision.

The BEKK approach (Engle and Kroner, 1995) was used to model market interdependencies, and the results of such modelling results were subsequently incorporated into the classic simplified portfolio selection model. The classic BEKK approach was further enhanced with Monte Carlo simulation for impulse response volatility and correlation (Allen et al., 2017). This allowed to opine on the data set length appropriate for the analysis as well as to capture the information content of BEKK modelled market dynamics into the portfolio selection problem.

The analysis is illustrated for a long-only Polish individual investors investing across four asset classes without possibility to hedge foreign exchange risk. It is based on market data from 2008–2020, additionally verifying the robustness of results during the COVID-19 related market turmoil.

The structure of the study follows the focus as defined above. Section 2 presents the decision problem concerning the construction of an individual investor's portfolio in the light of key assumptions. Section 3 discusses the dataset used together with its key characteristics, which is followed by the detailed discussion of the research methodology, including putting it in the context of earlier literature. Section 5 presents and discusses the key results with respect to market dynamics. Section 6 explains how they translate into portfolio selection decision. The concluding section summarises key findings and areas for further research.

## 2 Research focus and key assumptions

The study assumes the perspective of a Polish individual investor who is looking to optimize their long-term investment portfolio taking into account the rates of return in PLN. It is assumed that such an investor does not hedge any of its foreign investments (Driessen and Laeven, 2007) given limited availability of long-term FX hedges for individuals. Consequently, the rates of return on foreign assets are a combination of the rate of return of a given foreign asset and changes in the exchange rate.

The portfolio selection is optimized with the assumption of having only long exposures (short positions are not allowed) in four prime asset classes: Polish equities, Polish government bonds, foreign equities and foreign government bonds (euro area countries). The investor takes into account both the expected rates of return and risk (measured by the standard deviation). Portfolios are constructed to (with the limitations described above) maximize the Sharpe ratio[1].

Although using broadly defined asset classes, long-only investment approach and the lack of hedging may appear simplified approach, it makes the study results relevant for a

[1]Maximising Sharpe ratio is only one of the commonly used portfolio selection approaches (Farinelli et al., 2008). Preliminary calculations suggests that the main conclusions of this paper remained unchanged when using alternative portfolio optimisation approaches. However – in the interest of clarity – this method of portfolio selection has been consistently used in this study, while supplementary measures are expected to be used in further research.

wide range of individual investors, in line with the research objectives. Even an unsophisticated investor can easily access the considered portfolios based on the offer of mutual funds or exchange traded funds (ETFs), while complex hedging solutions are typically only available for qualified investors.

Another consciously accepted simplification is the assumption of no transaction costs. This allows to focus on the desired portfolio composition, rather than on the financial instruments or structures of accessing such exposure. That said, incorporating the transaction cost element would be necessary if the study results were to be applied to policy decisions on market design.

When analysing an optimal portfolio, the assumption about expected rates of return, volatility and interdependence are of critical importance. In this respect, the study contributes to the previous research by analysing an impact of taking various approaches towards estimating variance-covariance matrices, and therefore towards taking different methodological approaches towards incorporating market volatility and market interdependencies into the individual decision making process. Two alternative approaches are used towards defining the expected rates of return, to illustrate how such commonly used approaches may impact the results.

Market interdependencies remain in the centre of this study, therefore particular attention has been paid towards modelling the time-varying variance-covariance matrix of returns on financial assets. For this purpose, the study uses a diagonal version of the BEKK model (see the next section for more details).

Additionally, in order to determine the impact of uncertainty as to the model parameter estimates, the model estimation results for various time intervals were taken into account.

The analysis is conducted with the aim of addressing the following key research questions:

1. Does the commonly offered composition of balanced investment strategies (which include ca. 30% of equity) match the desired composition of a long-term individual investor?

2. To what extent is it optimal for a local currency focused investor to diversify onto foreign currency assets?

3. How sensitive is the optimal portfolio structure to different assumptions on the expected market returns or length of data series used?

4. How sensitive is the optimal portfolio structure to different assumptions underlying market independence?

# 3 Data used

As presented in the previous section, the study assumes a simplified choice between four distinct asset classes (equities and bonds, domestic or foreign), ignoring the transaction costs. Consequently, price indices representing particular asset classes were selected for the purposes of the study as follows. The Polish stock market is represented by the broad WIG index, the foreign stock market by the DAX index[2]. In the case of bonds, indices representative of the Polish treasury bond market in PLN (referred to as PL_BOND) and Euro-zone treasury bond market (referred to as EUR_BOND) were used, which account for both the changes in the valuation as well as coupon payments (total return indices). Consistently with the introduced assumption of local currency perspective, the rates of return on foreign assets were adjusted for euro-zloty exchange rate fluctuations.

The study covers the period from the beginning of 2008, when the behaviour of financial market participants began to show the first symptoms of the global financial crisis up until summer of 2020, thus capturing the financial market reaction to the first wave of the COVID-19 distortions. The snapshot on daily returns of the underlying time series is illustrated in Figure 1.

In line with the assumptions described earlier, the daily returns on foreign assets include the foreign exchange component. While this factor is relatively small in the case of equity series (given the fact that equity market volatility tends to exceed the commonly observed PLN volatility), it clearly dominates the daily returns on the foreign currency bonds, significantly increasing the perceived riskiness of this asset category. This is indirectly visible in Figure 1 when comparing the foreign exchange adjusted volatility of the EUR_BOND index to PL_BOND volatility, where the currency adjustment does not apply.

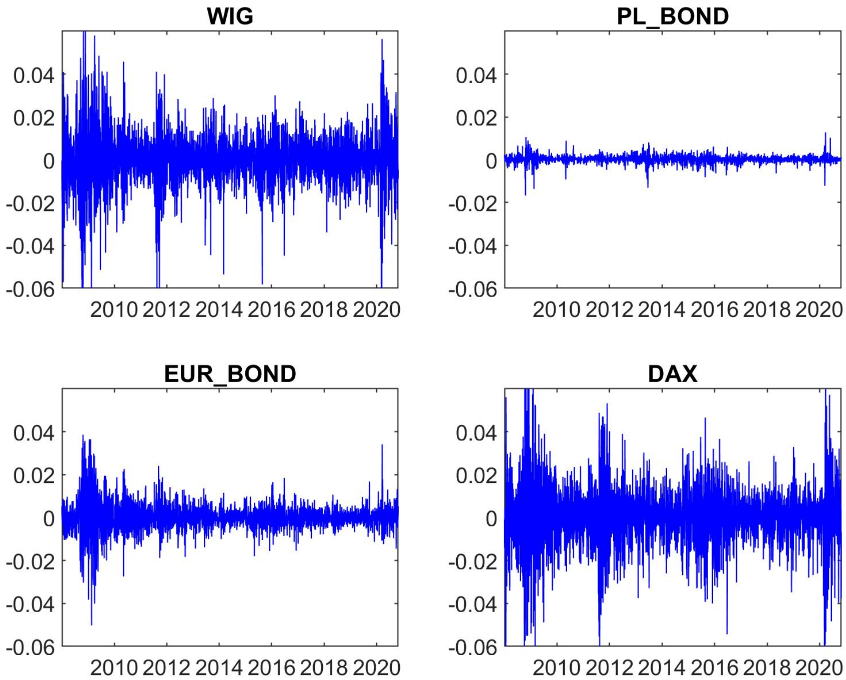
Figure 1 also clearly illustrates the time-varying nature of market volatility. Volatility is critical for risk assessment, therefore an investor facing a risk-return portfolio selection problem needs to take into account the time-varying nature of volatility when constructing and adjusting its portfolio. Therefore, it is reasonable to consider the use of GARCH conditional volatility models.

Many earlier studies (Tola et al., 2008) evidence that the distribution of daily returns in financial markets differs significantly between periods. Figure 2 and Figure 3 provide an illustration of the differences between the two subsamples of the analysed data.

The period between 2008 and 2012, following the outbreak of the global financial crisis,

[2] The use of DAX may appear non-intuitive at first. Alternative market indices were considered, e.g., an index of the global stock market or a portfolio index of several major stock exchanges. That said, given the focus on daily returns, the time zone alignment proved to play an important role in eliminating the time inconsistency of the data used. For example, the use of US index was leading to the problem of the same market signals being captured in different time observations, and thus leading to false interdependence matrices. This combined with very high correlation between the main stock indices (being in itself an evidence of growing popularity of wide diversification using passive investment strategy) triggered the choice of German index as most adequate for this study.

Figure 1: Daily returns of the analysed time series




Source: Author's own calculations.

is characterized by more frequent occurrences of extreme rates of return – and thus more visible "fat tails" in the return distributions. In the same way, market volatility decreases significantly when significant market distortions are avoided (e.g., years 2013–2019), to thereafter increase again towards the end of the sample (when COVID related distortions are observed).

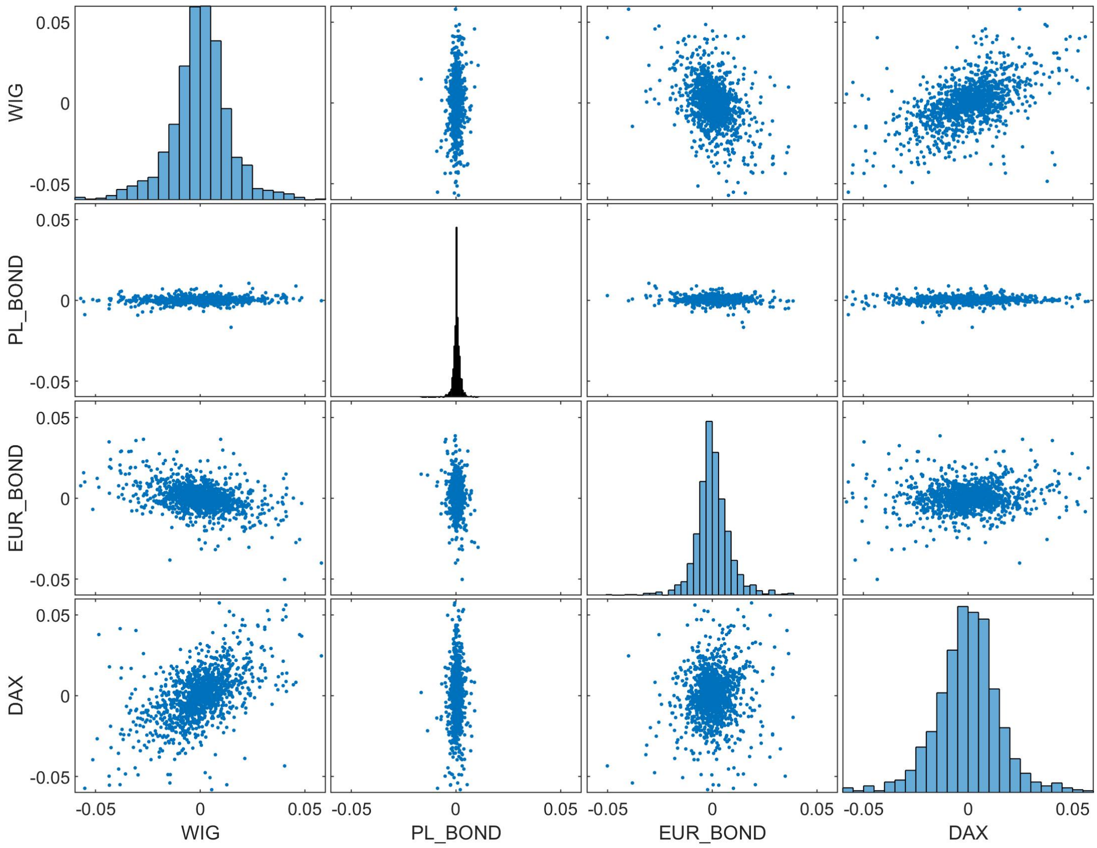
Apart from the volatility, the interdependence also changes its character between periods. A shift in the strength of the correlation can be seen when comparing Figure 2 and Figure 3. Interestingly, there is a small negative correlation between foreign bond and the Polish stock market, which signals a relatively strong role of foreign investors, who tend to exit the country during the market downturns.

The time varying nature of volatility and market interdependencies calls for using adequately flexible analytic tools. Therefore the following chapter discusses the details of the choice of modelling methodology in the context of prior research.

# 4 Detailed research approach on modelling market interdependencies in the context of earlier literature

This section discusses the methodology of estimating the variance-covariance matrix. Multivariate generalized autoregressive conditional heteroscedasticity (MGARCH) methods are widely used to model conditional volatility and the conditional correlation of finan-

Figure 2: Market returns and market interdependencies (2008–2012)




Source: Author's own calculations.

cial markets as well as the contagion effect. Both the literature review and the above mentioned preliminary data analysis justify the use of such approach, as it ensures capturing the time-varying character of the variance-covariance matrix. This approach is also consistent with literature (Pojarliev and Polasek, 2003; Tsiaras and Simos, 2020).

Of the many alternative detailed modelling specifications (e.g., Virbickaite et al., 2015; Bauwens et al., 2006; Silvennoinen and Teräsvirta, 2009), final one is often selected on the basis of technical constraints. This means that researchers tend to opt for the most flexible specification possible to be estimated with a given set of data. Since the most flexible specification is only possible with very long time series and narrow set of data, researchers use simplified specifications to reduce the number of estimated model parameters (Chang and McAleer, 2019).

The general form of the BEKK (Engle and Kroner, 1995) model $(1, 1, 1)$ can be presented as follows:

$$\Omega_t = C'C + A'\Omega_{t-1}A + B'r_{t-1}r'_{t-1}B \tag{1}$$

where:

$\Omega_t$ – variance-covariance matrix at time $t$;

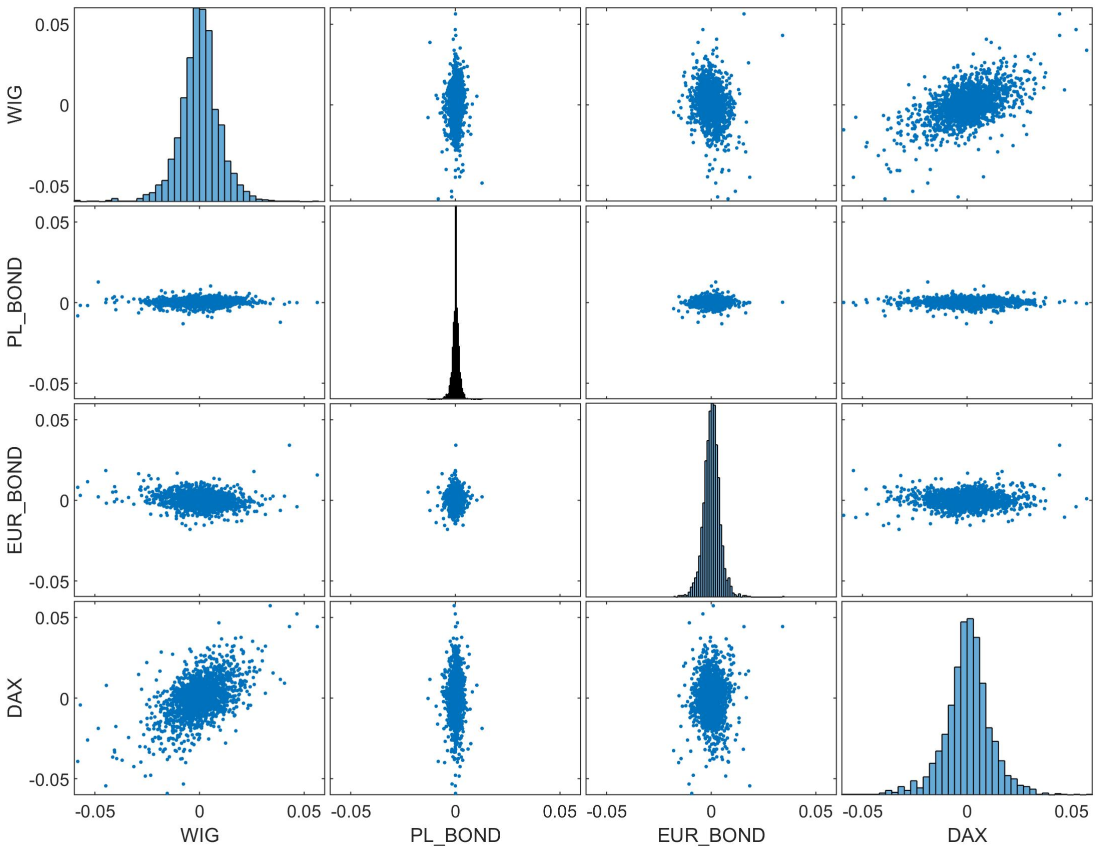
$A$, $B$, $C$ – matrices of parameters $(n \times n)$, where $C$ is an upper triangular matrix;
$n$ – number of assets;
$r_{t-1}$ – vector $(n \times 1)$ of filtered returns from $t-1$.

This generic specification requires long data series. In the case considered, it only turned possible to estimate such specification based on full sample (assuming a stable model dynamics over the entire period), while it was not possible for sub-periods. To allow to analyse changes in model dynamics, a diagonal version of the BEKK model was selected (where $A$ and $B$ are diagonal matrices).

The diagonal BEKK model is an informed choice over an alternative of DCC specification, which is also widely used in the financial markets literature (see for example: Caporin and McAleer, 2012; Aziz et al., 2019).

In the light of the covariance being at the very centre of interest of this study, DCC specification was assessed as too restrictive. The DCC analytical specification assumes shock transfer via the correlation of rates of return, which is considered a too far reaching simplification in the age of developed option markets, especially since there is significant amount of literature confirming contagion effects for volatility and covariance (e.g., Yousaf et al., 2020; Malik et al., 2021).

Figure 3: Market returns and market interdependencies (2013–2020)




Source: Author's own calculations.

The applied diagonal BEKK model is assumed to be a reasonable compromise between highly parameterised models, exposed to the "curse of dimensionality" and empirical tractability, still allowing for a possibility to capture indirect covariance contagion (Allen and McAleer, 2018; Chang and McAleer, 2019).

The selected specification was used to model the portfolio of four instruments (domestic and foreign equity and bonds). All time series were filtered using ARMA $(1,1)$. Toolbox of Oxford MFE (Sheppard, 2013) was applied to estimate model parameters.

Apart from the choice of modelling specification described above, the length of the data series to be used needs to be decided upon. The classic approach, which assumes model interdependencies – and consequently model parameters – to be stable over time – calls for using the longest possible, as they should produce the most robust results. In this case, however, taking too long time series (covering potentially periods of different model dynamics) creates a concern that the model estimated on long data series may not adequately reflect the interrelationships at a given point in time.

In this context, other research, including several studies on CEE markets (Chmielewska et al., 2018), indeed indicates high sensitivity of model outcomes depending on the length of the sample used to estimate such a model. This study confirmed that using somewhat shorter time series may still produce reliable estimates of market volatility and correlation. Therefore, for the purpose of this study, different time horizons were used: full sample and sub-sample approach. This helped to determine whether the different dynamics assumed when modelling variance-covariance matrices translated into similar or different portfolio selection decisions.

Given the fact that BEKK model parameters do not have a direct economic interpretation, it is more difficult to opine on the robustness of results (Allen and McAleer, 2018; McAleer, 2019). To that end, in line with recent literature, (Allen et al., 2017) simulated shock response analysis is proposed as robustness check and to better capture model dynamics.

The BEKK estimated unconditional variance-covariance matrix the facto corresponds to the long run ("equilibrium") and does not show interim model dynamics. To capture the latter, it is important to understand how the temporal variance-covariance matrix converges to this equilibrium after an incident of an imbalance. To analyse such a convergence process the BEKK model is supplemented by Monte Carlo simulation. According to Funke et al. (2022) and Gorgi et al. (2021) such "convergence path" or impulse response was simulated taking into account the values of the parameters of BEKK models estimated successively on a data sample. For the purpose of this paper, the simulation introduced three different volatility shock levels to the market volatilities (corresponding to 0.5 times, 1 times and 2 times the standard deviation respectively). 1000 model runs, covering 60 subsequent periods (days – representing 3 month period in our case), were run, allowing to get a quasi "impulse response" illustration.

In line with the main research question of this study, the BEKK modelling approach was incorporated into the simplified portfolio selection problem of an individual investor. Following Maller et al. (2010) and Dowd (2000), among others, it was assumed that the investor takes into account both the expected rate of return on investment of individual asset classes and the associated risk of the portfolio. The maximization of the Sharpe ratio was used to decide between alternative portfolio composition, investor's assumptions about the expected returns and variance covariance matrix allowed to calculate the composition of their optimal portfolio.

In this modelling, three alternative simplified methods of forming expectations as to future rates of return on financial assets, frequently used in the optimal portfolio literature, were used.

1. The actual 2013–2020 average daily returns of each asset class were taken as a time-constant assumption, i.e., assumption consistent with the concept of average forecasted returns consistent with efficient market hypothesis.

2. Historic average returns over a predefined period immediately preceding the portfolio adjustment decision. This implicitly reflects the assumption of continuation of market trends and is therefore criticised as suboptimal even in weakly efficient markets (Zielonka, 2004). In addition, it remains in line with the approach used by some of the market participants.

3. A constant equity risk premium, and thus time varying return expectations for equity, following the yield curve changes.

Four distinct approaches were used as assumptions with respect to the variance-covariance matrices.

1. The matrices implied by a full sample BEKK specification as a time-constant assumption.

2. The matrices implied by 2013–2020 BEKK model as a time-constant assumption.

3. The variance covariance matrices based on model re-estimation using the moving window of three year data immediately preceding the portfolio review.

4. A simulation based approach, which followed the approach discussed above for the GARCH model analysis. For each observation, the variance-covariance matrix was estimated based on the BEKK model. Such matrix, which determines the multidimensional distribution of possible returns in the next period, was taken as a starting point for the 60-day simulation. The ultimately used variance-covariance matrix was calculated based on cumulative returns of each of the 1000 model runs. Therefore, the resulting variance-covariance matrix for a three month period reflects both the starting point and the dynamics behind the estimated BEKK model.

Two distinct frequencies of adjusting the selected model composition were taken. Firstly, it was assumed that the portfolio structure is adjusted on a daily basis to reflect the optimal composition. While being technically possible in the light of no transaction cost assumption, such approach remains overoptimistic for individual investor. Therefore, quarterly portfolio updates were considered as the alternative.

# 5 Discussion of key interim results

## 5.1 Volatility and correlation modelling – preliminary findings

In line with the methodology choice explained in the previous section also the GARCH model was estimated multiple times using different length of the data inputs. Therefore each model was estimated for three data sets (i) full sample (2008–2020), (ii) 5 year period following Lehman turmoil (2008–2012) and (iii) recent period (2013–2020). The variance-covariance matrices we estimated using the following specification:

$$\widehat{\Omega}_t = \widehat{C}'\widehat{C} + \widehat{A}\widehat{\Omega}_{t-1}\widehat{A} + \widehat{B}r_{t-1}r'_{t-1}\widehat{B} \tag{2}$$

This, in turn, allowed to calculate volatility (standard deviation in annualised terms) for the underlying daily returns as well as correlation between the asset classes for the period between 2013 and 2020.

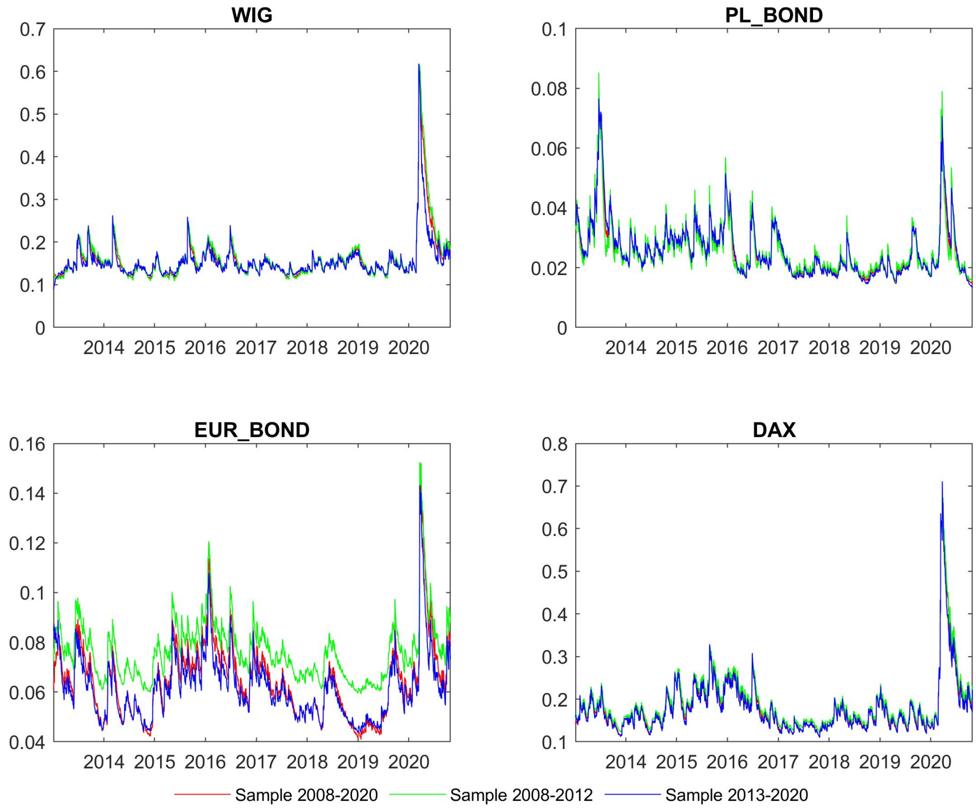
The results, presented in Figure 4 and Figure 5, suggest a relatively comparable outputs across all three specifications. At the same time, it is visible that the full sample model results in higher volatility.

This means that had the investor estimated model parameters based on 2008–2012 sample and had they used such specification in unadjusted manner in the subsequent period, they would have overestimated current volatility as opposed to the factual one. This finding clearly calls for a need for periodic updates of the model specifications, as the new data become available.

On high level findings, these interim results reconfirm low volatility of returns for domestic bond portfolio as well as a relatively low correlation of this asset class with the remaining ones. Intuitively, most robust correlation can be observed between the two stock exchange indices, which is additionally reinforced in the periods of increased market volatility. This means that in those higher risk periods (higher volatility of financial assets), i.e., when investors seek risk reduction techniques, the risk mitigation mechanism through asset diversification for equity markets is significantly weaker.

Moreover, a wide range of correlation estimates is worth mentioning. For instance, the correlation between the domestic and foreign bonds ranges from -0.5 to +0.5, while between the two stock markets, it ranges from 0.2 to 0.8. This questions the approach in which an investor uses a single correlation number and thus calls for further guidance as to using such model results in practice.

Figure 4: Volatility estimates – full sample and sub-sample results



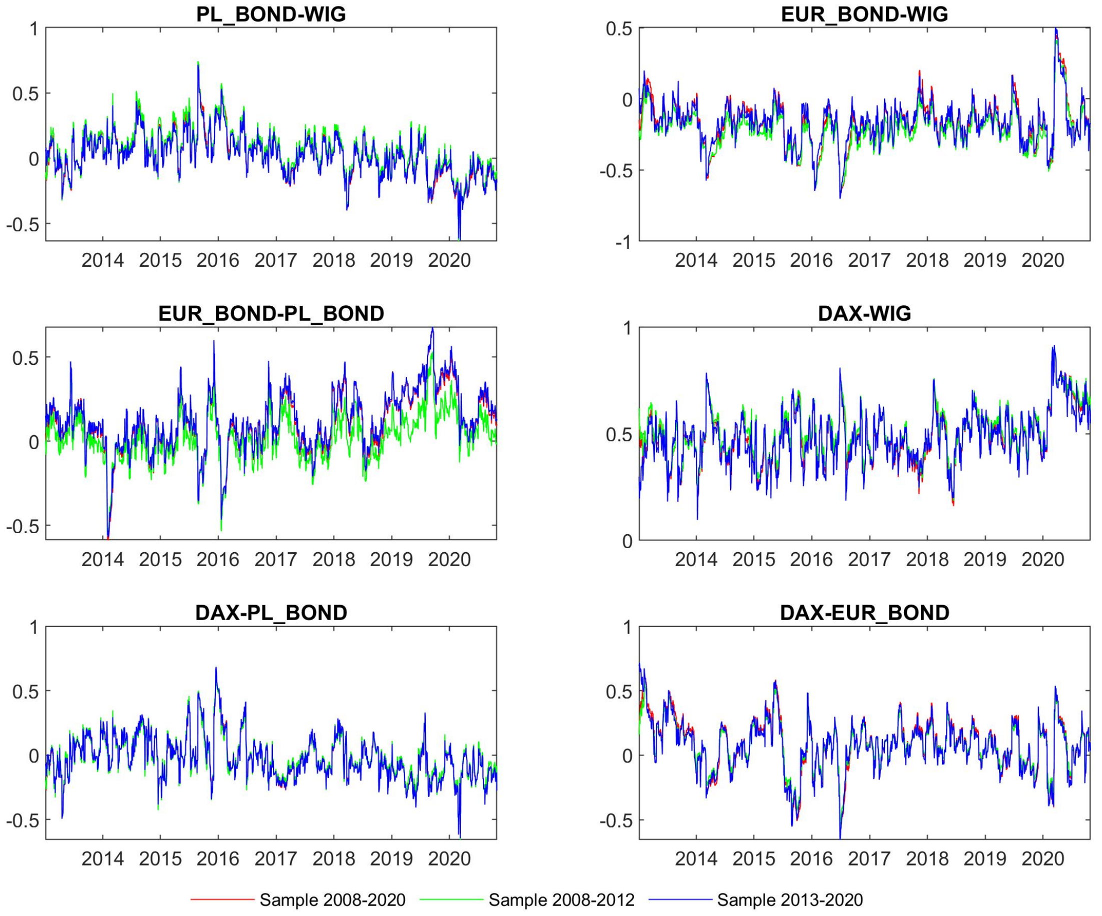

Source: Author's own calculations.

## 5.2 Dynamics character of volatility and correlation – simulation results

While the estimation results presented above appear to be relatively close between alternative specifications, the estimated variance-covariance matrix does not yet fully capture the characteristics of the estimated conditional variability models (BEKK). Most importantly, they do not provide information on the dynamic features of these models.

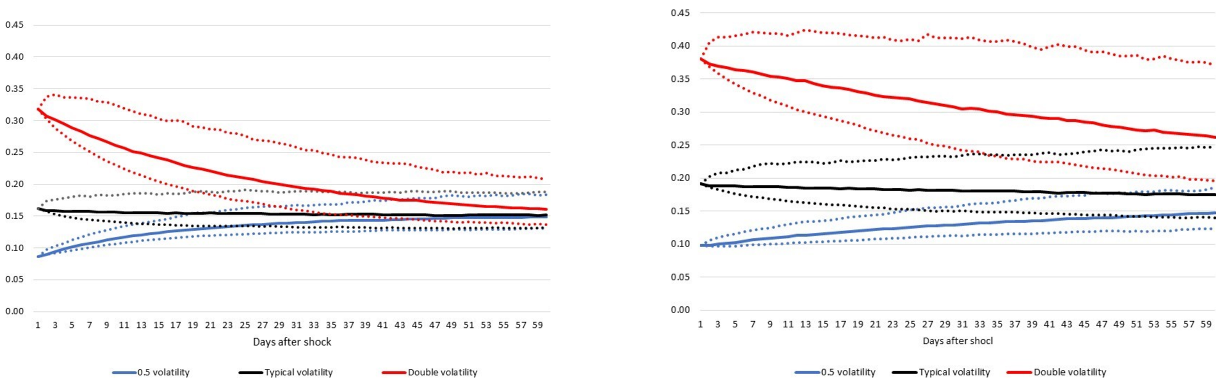
At the same time, from the perspective of an investor striving for optimal portfolio selection, it is critical to understand the trajectory of changes in estimated volatility and correlation over the portfolio maintenance horizon, and specifically after the portfolio is hit by an unexpected event. Therefore, an informed investor might be interested in the "convergence path", which, as explained in the methodology section, was simulated over 3 months period for three different shock levels. This was achieved by taking into account the values of the parameters of BEKK models estimated successively on a full sample (2008–2020) as well as the more recent part of this period (2013–2020). The results of this simulation are presented in Figure 6, where the thick line shows the average level across all model runs, while the dotted lines represent 5$^{\text{th}}$ and 95$^{\text{th}}$ centile.

Figure 5: Selected correlation estimates – full sample and subsample model results




Source: Author's own calculations.

Figure 6: Impulse response in respect of volatility shift on WIG, based on 2013–2020 model (left hand side) and full sample model (right hand side)




Source: Author's own calculations.

This approach evidences a much higher dispersion of the convergence path based on the full sample. As expected, that market dynamics might be completely different between periods. Higher volatility average in the full sample compared to the more recent part translated not only to the higher estimated equilibrium level, but also resulted in higher uncertainty of results and longer convergence period.

The results presented in this part of the study lead to the conclusion that in the event of significant differences in market dynamics within the observed period, increasing the number of observations used in estimation by extending the sample may hinder the accuracy of estimates especially outside of the "quasi equilibrium" state.

In the following part of the study, the author will try to answer whether the different model dynamics can translate into visibly different economic decisions. Therefore, further part of the study will try to determine whether using different model specifications can translate into visibly different portfolio composition.

## 6 Portfolio selection – selected results

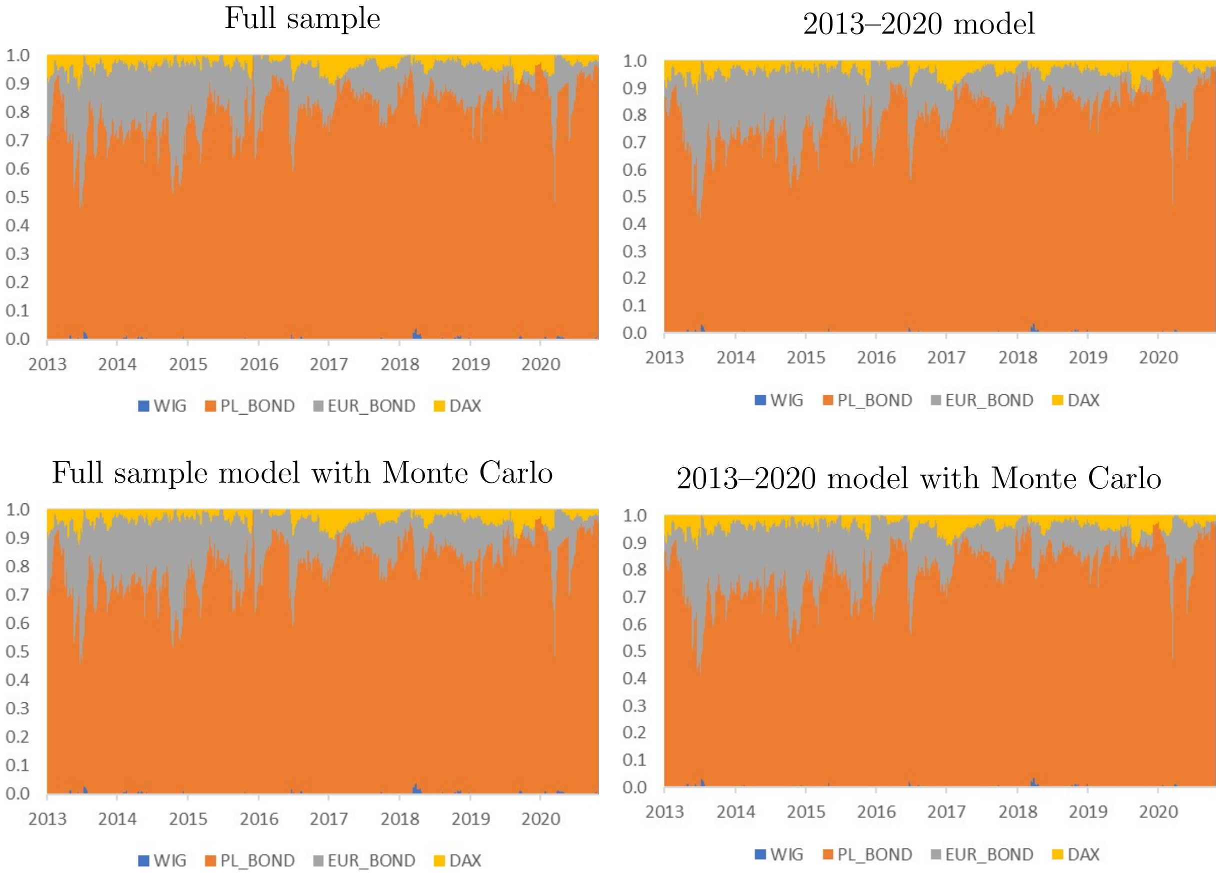
This section discusses the portfolio selection results obtained by using the different modelling approaches.

Figure 7 illustrates portfolio selection of an investor using a constant return expectations. The graphs in the upper section reflect the portfolio choice when directly applying the BEKK model, while the graphs in the lower part incorporate the additional simulation approach. In each case, the results are illustrated for a full and recent sample estimation respectively.

Although the presented results appear qualitatively similar, some differences, which can be attributed to the model dynamics assumptions, are clearly visible. Direct use of BEKK model estimates leads to lower share of domestic assets (shares and bonds in total) during the increased volatility periods. The use of simulation approach leads to more stable portfolio structure and does not react with overweighting foreign investment during the market uncertainty. As expected, the difference between portfolios based on different sample length is more visible when using the Monte Carlo approach. An interesting conclusion is the fact that using more recent data translates to a higher share of foreign bonds in the specifications. This is consistent with the earlier hypothesis that the use of the full sample may overestimate the riskiness of foreign exchange component connected with this asset class, and thus making it a better substitute for equity component.

Figure 8 presents an equivalent set of results for an investor who regularly updates his return expectations based on yield curve and arbitrarily selected 5% equity premium (largely consistent with multiple brokerage research). The main conclusion of more stable portfolio structure when using extended data sample holds. Similarly, when using more recent data set, investors seem to decide to keep higher share of risky assets, which, again, reflects the earlier hypothesis that data sets which include turmoil periods lead to more

Figure 7: Portfolio selection results – alternative correlation modelling approaches under the constant return expectation model

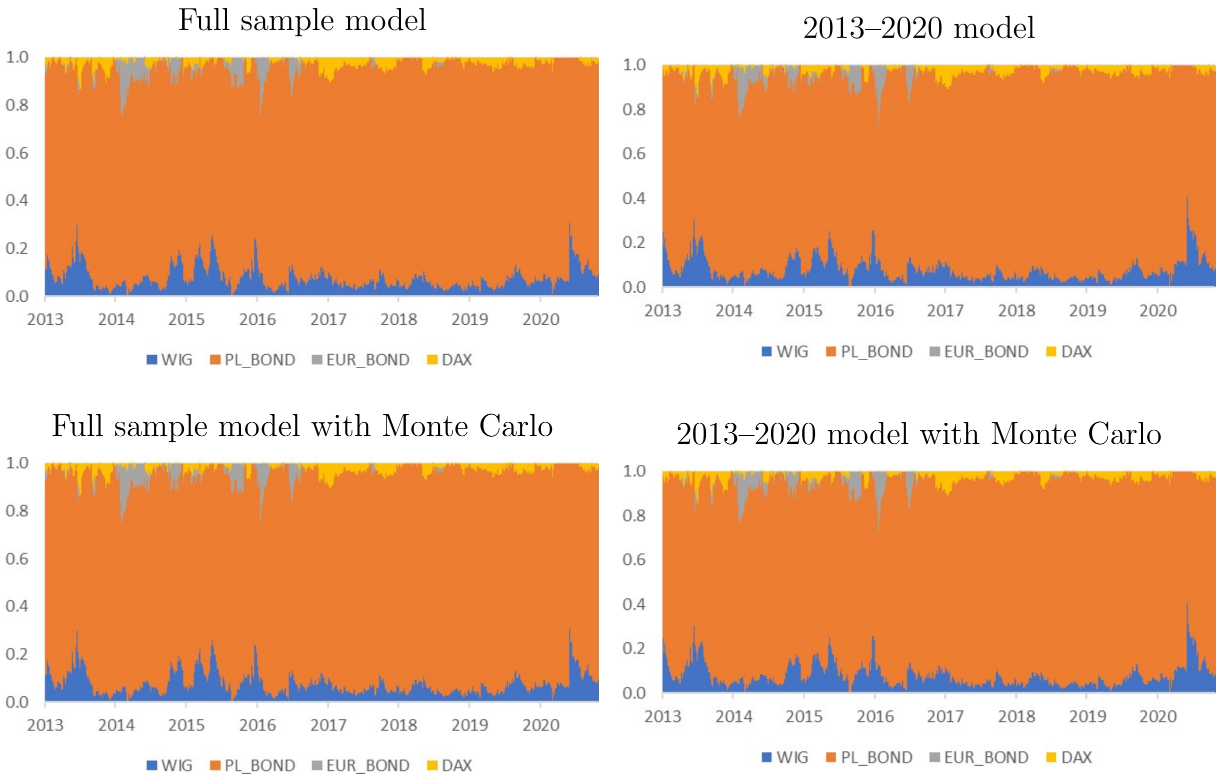



Source: Author's own calculations.

conservative portfolio selection decisions. Compared to Figure 7 however, all portfolios are characterized by a higher share of domestic equities, with a noticeably smaller role of investments in foreign bonds. This reflects a natural choice of equity to foreign exchange risk factor, where the latter does not assume a risk premium. Consequently, the risk premium assumption results in an increased home-country bias and therefore challenges the rationale for a wider international diversification.

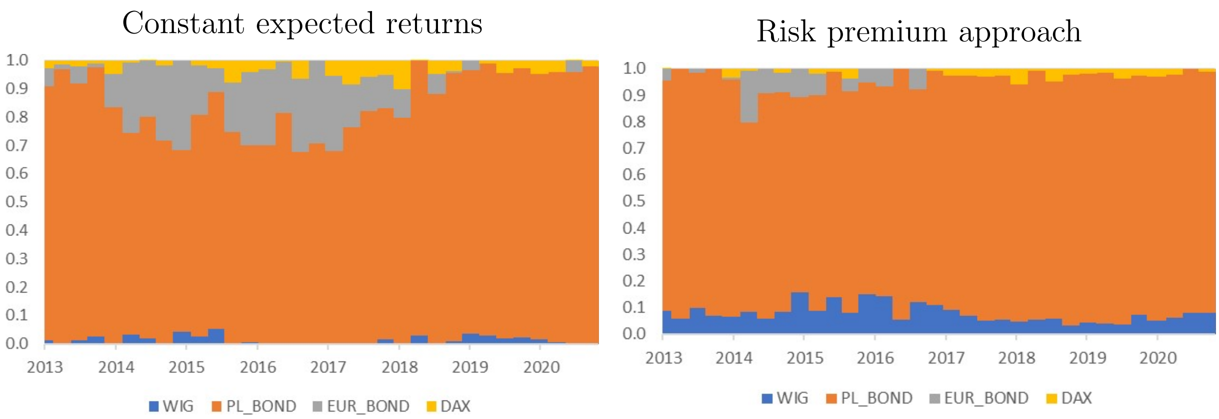
Figure 9 illustrates portfolio selection approach based on the BEKK model quarterly updated on a sliding three-year window. This represents an investor who wants to have a strategy consistent with the last three years of market conditions. The first graph in Figure 9 illustrates the portfolio selection using average returns approach for return expectations, while the second chart reflects the risk-premium approach. It is clearly visible that the application of momentum strategy elements (i.e., the use of a historical rate of return – see, e.g., Rachev et al. (2007)) causes significant fluctuations in the portfolio structure. Limited reliance on historic data with respect to returns results in greater stability of the portfolio structure. Moreover, trust in the existence of a risk premium clearly incentivises wider use of equity.

Figure 8: Portfolio selection results – alternative correlation modelling approaches under time varying return expectations




Source: Author's own calculations.

Figure 9: Portfolio selection results – alternative correlation modelling approaches under time varying return expectations




Source: Author's own calculations.

Different model specifications are summarised in Table 1 and Table 2. Table 1 shows the average (annualized) rate of return, volatility and the Sharpe ratio, while Table 2 presents the average portfolio composition (together with its standard deviation).

The following conclusions can be drawn on the basis of the above results.

Table 1: Portfolio assumptions and key characteristics

| Estimation sample | Expected returns | Var-covar matrix | Return rate (annualised) | Volatility (annualised) | Sharpe ratio | Figure |
|---|---|---|---|---|---|---|
| 2008–2020 | 2013–2018 average | BEKK | 4.570% | 2.616% | 1.747 | Figure 7 (1) |
| 2013–2020 | 2013–2018 average | BEKK | 4.570% | 2.630% | 1.738 | Figure 7 (2) |
| 2008–2020 | 2013–2018 average | BEKK and MC | 4.555% | 2.580% | 1.766 | Figure 7 (3) |
| 2013–2020 | 2013–2018 average | BEKK and MC | 4.574% | 2.631% | 1.738 | Figure 7 (4) |
| 2008–2020 | Risk premium | BEKK | 3.987% | 3.009% | 1.325 | Figure 8 (1) |
| 2013–2020 | Risk premium | BEKK | 4.269% | 3.115% | 1.370 | Figure 8 (2) |
| 2008–2020 | Risk premium | BEKK and MC | 3.936% | 3.008% | 1.309 | Figure 8 (3) |
| 2013–2020 | Risk premium | BEKK and MC | 4.277% | 3.284% | 1.302 | Figure 8 (4) |
| 3-year rolling | Average for estimation sample | BEKK | 3.876% | 2.651% | 1.462 | Figure 9 (1) |
| 3-year rolling | Risk premium | BEKK | 3.894% | 2.855% | 1.364 | Figure 9 (2) |

Source: Author's own calculations.

Table 2: Portfolio composition

| | | | WIG | | PL_BOND | | EUR_BOND | | DAX | |
|---|---|---|---|---|---|---|---|---|---|---|
| Estimation sample | Expected returns | Var-covar matrix | Share | Std. dev. | Share | Std. dev. | Share | Std. dev. | Share | Std. dev. |
| 2008–2020 | 2013–2018 average | BEKK | 0.23% | 0.60% | 82.10% | 9.26% | 14.49% | 9.27% | 3.17% | 2.35% |
| 2013–2020 | 2013–2018 average | BEKK | 0.21% | 0.58% | 81.38% | 9.95% | 15.04% | 9.89% | 3.37% | 2.44% |
| 2008–2020 | 2013–2018 average | BEKK and MC | 0.06% | 0.28% | 82.75% | 5.84% | 14.19% | 5.87% | 3.00% | 1.52% |
| 2013–2020 | 2013–2018 average | BEKK and MC | 0.05% | 0.28% | 80.99% | 7.53% | 15.66% | 7.29% | 3.30% | 1.59% |
| 2008–2020 | Risk premium | BEKK | 8.37% | 5.30% | 88.07% | 6.36% | 1.57% | 3.72% | 1.99% | 2.15% |
| 2013–2020 | Risk premium | BEKK | 8.83% | 5.88% | 87.53% | 7.05% | 1.51% | 3.78% | 2.14% | 2.28% |
| 2008–2020 | Risk premium | BEKK and MC | 8.30% | 4.17% | 89.11% | 4.72% | 0.84% | 2.41% | 1.75% | 1.46% |
| 2013–2020 | Risk premium | BEKK and MC | 9.36% | 5.75% | 87.98% | 6.55% | 0.73% | 2.38% | 1.93% | 1.57% |
| 3-year rolling | Average for estimation sample | BEKK | 1.24% | 1.46% | 83.65% | 10.69% | 11.90% | 10.57% | 3.20% | 2.66% |
| 3-year rolling | Risk premium | BEKK | 7.78% | 3.29% | 87.98% | 6.01% | 2.57% | 4.45% | 1.67% | 1.58% |

Source: Author's own calculations.

Firstly, the highest Sharpe ratios were obtained for the portfolios for which the expected rates of return were set as the actually realized average for the entire period. Unfortunately obtaining such insight into future returns is not possible in practice. Similarly, taking into account the relatively weak profitability of equity investments in the analysed period, it is not surprising that the share of WIG in portfolios based on historical rates of return is lower when compared to portfolios with a premium for equity market risk. In the former case, risky assets are replaced by foreign assets, where the foreign exchange risk factor de facto substitutes the equity risk factor.

Given the fact that the focus of this paper is on the impact of different approaches to interdependence modelling, it is worth underlying a couple of additional findings. Main counter-intuitive finding is that models estimated for a shorter sample lead to obtaining portfolios that are more favourable from the investor's point of view (characterized by a higher Sharpe ratio). Therefore, a very careful consideration when determining sample length is highly recommended. Modelling benefits of longer sample tend to be outweighed by regularly updates shorter sample models. This suggests that structural changes taking place in financial markets (such as existence of volatility and correlation clustering) create situations where the distant observations do not necessarily have a positive effect on the quality of model estimates, and can hinder the effectiveness of the investor's decision-making process. Alternative solution of adjusting the weights of more remote data points, could be explored in further research.

It is also worth noting that the additional information about the market dynamics, captured in the simulated approach, translate into higher Sharpe ratio. This indirectly confirms that information about the dynamics of volatility and asset correlation can improve the efficiency of decision making. This constitutes yet another argument in favour of using multi-dimensional conditional volatility models in the investment process.

# 7 Concluding remarks

This paper presents the portfolio selection problem with the use of multivariate conditional volatility models. This research has both methodological and practical implications.

Various approaches towards capturing the model interdependencies were applied. Different specifications resulted in relatively similar estimates of variance-covariance matrix. However, enhancing methodology with the use of Monte Carlo simulation uncovered the importance of the assumed model dynamics. Additional information content captured by such an approach, allowed to optimize further the portfolio selection decision making process, ultimately translating into higher Sharpe ratios.

Different modelling approaches also provided a convincing argument against the use of very long time series for a regularly updated portfolio selection decision. The volatility and correlation clustering encourage regular adaptation of market dynamic assumptions alongside with recent market conditions. Unreflective extension of the sample, which may appear sound from a pure modelling perspective, does not necessarily lead to model quality improvement.

On the portfolio composition front, the results provide a powerful argument in favour of home currency bias and highly conservative investment strategy for an individual investor who is interested in local currency returns and cannot benefit from currency hedging. Very clear dominance of domestic bond portfolio proves robust regardless of the modelling approaches.

The share of foreign assets is highly dependent on the underlying equity return expectations. This indirectly confirms the fact that for a local currency investor, the use of foreign assets is de facto an integral part of the risky component of the underlying portfolio. The addition of such new risk factor is only justified, if the connected risk premia and diversification gain is not significantly superior to risk asset classes available in the local market.

Interestingly, in all approaches, the resulting share of government bonds significantly exceeds levels commonly offered by mutual funds or recommended by capital market oriented pension systems. This finding suggests that, at least for PLN investor, the higher expected rates of return for risky assets do not always compensate for the excess risk related to such investments. This calls for a wider discussion on the design on the pension system, which shall not excessively transfer economic risk onto individual investors.

The findings of the study support a relatively low share of foreign assets in the structure of savings of individual investors. This contrasts with the directly applied theory of passive investment. This appears to be largely driven by the absence (from the perspective of an individual investor) of an effective tool for eliminating currency risk and the assumed investors interest in local currency returns. For such an investor, excessive exposure to foreign assets would significantly deteriorate the portfolio parameters.

## 8 Further research areas

To the author's best knowledge, this research is the first use of impulse response as enhancement for BEKK approach to volatility and correlation in modelling of individual investment decisions in the Polish context. Therefore, it would be beneficial for further research to verify these results. The decision making process can be enhanced by using different optimization approaches and analysis of the optimal portfolio update frequency. It would be interesting to verify whether the portfolio composition would be closer to classic passive investor approach, provided it was assumed that an individual investor optimises purchasing power rather than local currency return (as assumed in the paper).

Moreover, the use of alternative asset categories need to be explored. The incorporation of liquidity premia and transaction costs may help explain the prevailing strong bias towards classic bank deposits in the private savings. Short-term currency hedge possibility or the use of equity futures could also be explored in order to validate the robustness of the local currency bias from the perspective of a more informed individual investor.

On methodology front, wider data sets, covering other countries and more asset classes, may be used to verify the robustness and finetune the methodology suggestions.

In the Polish context, further research would be welcomed to determine whether the portfolio selection decision shall take into account the length of the pension savings period (which remains outside of scope of this paper). This might help to opine whether it is indeed justified for a younger pension plan participant to lean towards more risky investment strategies or whether alternative mitigants (rather than a gradually changing investment strategy) of exit valuation risk could be explored.

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